Solutions to Chapter 4:

Exercise 4.1: Recombination in the c-Si Solar Cell

a) The most important ones are imperfection recombinations (foreign atoms, crystal structure errors) and surface recombinations.

b) This ist he upper part oft he n^+-emitter layer. Because of the high doping concentration the produced holes recombine very quickly. Therefore this region contributes hardly to the photo current.

c) $x_{\mathrm{Abs}} = 140$ µm, $\tau_{\mathrm{N}} = 7$ µs,

$$L_{\mathrm{N}} = \sqrt{D_{\mathrm{N}} \cdot \tau_{\mathrm{N}}} = \sqrt{35 \frac{\mathrm{cm}^2}{\mathrm{s}} \cdot 7 \cdot 10^{-6}\mathrm{s}} = 156.5\ \mu\mathrm{m} > x_{\mathrm{Abs}}$$

Yes, the generated electron is likely to contribute to the photo current.

Exercise 4.2: Absorption Efficiency of a c-S Cell

a) $x_{\mathrm{P}} = 1/\alpha = 1\ \mathrm{cm} / 100 = \underline{100\ \mu\mathrm{m}}$

b) $R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 = \left(\frac{1-3.3}{1+3.3}\right)^2 = \underline{28.61\ \%}$

$E_{\mathrm{R}} = R \cdot E_0 = \underline{286.1\ \mathrm{W/m^2}}$

c) $E_1 = E(x=0) = (1-R) \cdot E_0 = \underline{713.9\ \mathrm{W/m^2}}$

$E_2 = E(x=d) = E_1 \cdot e^{-\alpha \cdot d} = \underline{176.0\ \mathrm{W/m^2}}$

$E_{\mathrm{Abs}} = E_1 - E_2 = \underline{537.9\ \mathrm{W/m^2}}$

d) With the mirrored rear side the optical cell thickness is practically doubled:

$E_1 = E(x=0) = E_0$

$E_2 = E(x = 2 \cdot d) = E_1 \cdot e^{-\alpha \cdot 2 \cdot d} = \underline{60.8\ \mathrm{W/m^2}}$

$E_{\mathrm{Abs}} = E_1 - E_2 = \underline{939.2\ \mathrm{W/m^2}}$

e) $\eta_{\mathrm{Abs}} = \frac{E_{\mathrm{Abs}}}{E_0} = \underline{93.22\ \%}$

$S(\lambda) = \frac{q}{h \cdot c} \cdot \lambda \cdot \eta_{\mathrm{Ext}}$

$\text{Here: } \eta_{\mathrm{Ext}} = \eta_{\mathrm{Abs}} \Rightarrow S(\lambda) = \frac{1}{1{,}24\ \mu\mathrm{m}} \cdot 1\ \mu\mathrm{m} \cdot 93.22\ \% = \underline{75.18\ \%}$

Exercise 4.3: Single Diode Equicalent Circuit Diagram

a) See Section 4.5.2

b) Series Resistance:
Resistance of the contact fingers on the top side of the cell, metal-semiconductor junction resistance, ohmic resistance in the semiconductor volume.

Shunt Resistance:
Local short circuits of the p-n junction, insufficient insulation at the edges of the solar cell.

c) See Figure 4.15 a)
In the short circuit point the current sinks with rising R_S. The reason is that there is a rising voltage drop at R_S which leads to a Rising V_D and thus a rising diode current.
The open circuit point will not change as here no current flows through R_S and therefore no voltage drops at it.

d) See Figure 4.15 b)
With falling R_P in the open circuit point the portion of photo current flowing trough R_P will rise. For the diode there will be less current available thus V_D becomes smaller. As in the open circuit point the condition $V_D = V_{OC}$ holds, V_{OC} will decrease with falling R_P.
The short circuit point is almost independent of R_P as in this case R_P and R_S are connected in parallel and typically R_S is one order of magnitude smaller than R_P.

Exercise 4.4: Spectral and Theoretical Efficiency

a) The spectral efficiency denotes which part of the incident radiation power can be used theoretically with a semiconductor of bandgap ΔW_G. The transmission losses describe the part of the spectrum where the photons have not enough energy to overcome the bandgap. Thermalization on the other hand arises with photons that have more energy than the bandgap. In this case only a part of the energy can be used for the cell, the rest is converted into heat.

b) The theoretical efficiency accounts additionally two loss sources: On the one hand in a solar cell not the whole voltage can be used $\Delta W_G/q$. On the other hand the use of a p-n junction causes that the fill factor is always smaller of 100 %.

c) The theoretical efficiency of c-Si cells is about 28.6 %. The world record cell from Australia lies very close to this optimum with its measured efficiency of 25 %.

Exercise 4.5: Spectral Efficiency for Monochromatic Light

a) $$N_{\mathrm{Ph}} = \frac{W_{\mathrm{Opt}}}{W_{\mathrm{Ph}}} = \frac{P_{\mathrm{Opt}} \cdot \Delta t}{h \cdot f} = \frac{E \cdot A \cdot \Delta t}{\frac{h \cdot c}{\lambda}} = \frac{1000\ \mathrm{W/m^2} \cdot 10\ \mathrm{cm^2} \cdot 1\ \mathrm{s}}{\frac{6.6 \cdot 10^{-34}\,\mathrm{W \cdot s^2} \cdot 3 \cdot 10^{8}\,\mathrm{m/s}}{1000 \cdot 10^{-9}\,\mathrm{m}}} = \underline{5.051 \cdot 10^{19}}$$

$$j_{\mathrm{Max}} = \frac{\text{charge}}{\text{time} \cdot \text{area}} = \frac{N_{\mathrm{Ph}} \cdot q}{\Delta t \cdot A_{\mathrm{Cell}}} = \frac{5.051 \cdot 10^{19} \cdot 1.6 \cdot 10^{-19}\,\mathrm{As}}{1\ \mathrm{s} \cdot 10\ \mathrm{cm^2}} = \underline{80.8\ \mathrm{mA/cm^2}}$$

b) Saturation current density: $j_{\mathrm{S}} = K_{\mathrm{S}} \cdot e^{-\frac{\Delta W_G}{k \cdot T}} = 40\,000\ \mathrm{A/cm^2} \cdot \mathrm{e}^{-\frac{1.12\,\mathrm{eV}}{8.63 \cdot 10^{-5}\,\mathrm{eV/K} \cdot 298.15\mathrm{K}}} = 5\ \mathrm{fA/cm^2}$

Open circuit voltage: $V_{\mathrm{OC}} = V_{\mathrm{T}} \cdot \ln \frac{I_{\mathrm{SC}}}{I_{\mathrm{S}}} = V_{\mathrm{T}} \cdot \ln \frac{j_{\mathrm{Max}}}{j_{\mathrm{S}}} = 26\ \mathrm{mV} \cdot \ln \frac{80.8\ \mathrm{mA/cm^2}}{5 \cdot 10^{-15}\,\mathrm{A/cm^2}} = \underline{791\ \mathrm{mV}}$

c) With Equation (4.18): $$FF = 1 - \frac{1 + \ln(\frac{V_{\mathrm{OC}}}{V_{\mathrm{T}}} + 0.72)}{\frac{U_{\mathrm{L}}}{U_{\mathrm{T}}} + 1} = 1 - \frac{1 + \ln(\frac{791\ \mathrm{mV}}{26\ \mathrm{mV}} + 0.72)}{\frac{791\ \mathrm{mV}}{26\ \mathrm{mV}} + 1} = \underline{85.9\ \%}$$

d) With Equation (4.48): $$\eta_{\mathrm{T}} = \frac{V_{\mathrm{OC}} \cdot j_{\mathrm{Max}} \cdot FF}{E} = \frac{0.791\ \mathrm{V} \cdot 80.8\ \mathrm{mA/cm^2} \cdot 0.859}{1000\ \mathrm{W/m^2}} = \underline{54.9\ \%}$$

e) $j'_{\mathrm{Max}} = j_{\mathrm{Max}} \cdot 1000 = \underline{80.8\ \mathrm{A/cm^2}}$

$$V_{\mathrm{OC}} = V_{\mathrm{T}} \cdot \ln \frac{1000 \cdot j_{\mathrm{Max}}}{j_{\mathrm{S}}} = V_{\mathrm{OC}} + V_{\mathrm{T}} \cdot \ln 1000 = 791\ \mathrm{mV} + 26\ \mathrm{mV} \cdot 6.908 = \underline{971\ \mathrm{mV}}$$

$$FF' = 1 - \frac{1 + \ln(\frac{V'_{\mathrm{OC}}}{V_{\mathrm{T}}} + 0.72)}{\frac{V'_{\mathrm{OC}}}{V_{\mathrm{T}}} + 1} = 1 - \frac{1 + \ln(\frac{971\ \mathrm{mV}}{26\ \mathrm{mV}} + 0.72)}{\frac{971\ \mathrm{mV}}{26\ \mathrm{mV}} + 1} = \underline{87.9\ \%}$$

$$\eta'_{\mathrm{T}} = \frac{V'_{\mathrm{OC}} \cdot j'_{\mathrm{Max}} \cdot FF'}{E'} = \frac{0.971\ \mathrm{V} \cdot 80.8\ \mathrm{A/cm^2} \cdot 0.879}{1\,000\,000\ \mathrm{W/m^2}} = \underline{68.9\ \%}$$