Solutions to Chapter 2:

Exercise 2.1: Solar Constants

a) $E_S \sim \frac{1}{r_{SE}^2} \Rightarrow E_S' = E_S \cdot (\frac{r_{SE}}{r_{SE}'})^2$

$$E_{S\_Max} = E_S \cdot (\frac{r_{SE}}{r_{SE\_Min}})^2 = 1367 \ \frac{W}{m^2} \cdot (\frac{149.5}{147})^2 = 1413.9 \ \frac{W}{m^2}$$

$$E_{S\_Min} = E_S \cdot (\frac{r_{SE}}{r_{SE\_Max}})^2 = 1367 \ \frac{W}{m^2} \cdot (\frac{149.5}{152})^2 = 1322.4 \ \frac{W}{m^2}$$

b) $E_{S\_Mercury} = E_S \cdot (\frac{r_{SE}}{r_{SM}})^2 = 1367 \ \frac{W}{m^2} \cdot (\frac{149.5}{58})^2 = 9.082.3 \ \frac{W}{m^2}$

Exercise 2.2: Solar Spectrum

a) AM 0 is the spectrum of the Sun outside the atmosphere of the Earth.

b) AM 1.5 is the spectrum of Sun rays, which results after rays travelling through a path length of 1.5 times the thickness of the atmosphere. The Sun height angle is calculated to:

$$1{,}5 = \frac{1}{\sin \gamma_S} \Rightarrow \gamma_S = \arcsin(\frac{1}{1{,}5}) = \underline{41.8\,°}$$

c) In the atmosphere light is scattered at particles (e.g. single molecules) which are smaller than the wavelength of the light. This Rayleigh scattering has a strong wavelength dependence of $R \sim 1/\lambda^4$. Blue light (short wavelength) is therefore scattered stronger than red light.

d) In the case of afterglow the observer looks in the direction of Sun which is low. The white light of the Sun therefore travels a long way through the atmosphere up to the observer. Thereby especially the blue light is scattered away to the sides by Rayleigh scattering. Left over is mainly the red light.

Exercise 2.3: Global Radiation

a) The diffuse radiation is generated by scattering of the sunlight at the atmosphere . Here the relevant effects are Rayleigh scattering at small particles (e.g. single air molecules) and Mie scattering at larger particles (e.g. aerosols and dust particles).

b) Depending on the site the portion is 54 to 60 %; roughly spoken slightly more than half.

c) The Sun full load hours denote the number of hours the Sun has to shine with full power (E = 1000 W/m^2) to generate the real annual radiation sum. In Germany the Sun full load hours on a horizontal surface is about 1000 h.

d) From Table 2.4 we see that over a year a horizontal surface receives only about 86.4 % of the radiation energy at optimum slope angle. Therefore the Sun full load hours for a horizontal surface are:

$$1000 \text{ h/a} \cdot \frac{1}{0.883} = \underline{1157.4 \text{ h/a}}$$

Exercise 2.4: Radiation on Tilted Surfaces

a) Optimum angle at β_{Opt} = 90 ° - γ_S = 40 °

$$E_{Gen\_Max} = E_{Direct\_H} \cdot \left(\frac{\sin(\gamma_S + \beta)}{\sin(\gamma_S)}\right) = 850\ \frac{W}{m^2} \cdot \left(\frac{\sin(90°)}{\sin(50°)}\right) = \underline{1109.6\ \frac{W}{m^2}}$$

b) $$E_{Gen} = E_{Direct\_H} \cdot \left(\frac{\sin(\gamma_S + \beta)}{\sin\gamma_S}\right) = 850\ \frac{W}{m^2} \cdot \left(\frac{\sin 65°}{\sin 50°}\right) = \underline{1005.6\ \frac{W}{m^2}}$$

c) γ_S = 25 °, $E_{Direct\_H} = E_{Diffus\_H}$ = 300 W/m²

$$E_{Gen} = E_{Direct\_H} \cdot \left[\frac{\sin(\gamma_S + \beta)}{\sin\gamma_S} + \frac{1}{2}(1 + \cos\beta)\right] = 300\ \frac{W}{m^2} \cdot \left[\frac{\sin(25° + \beta)}{\sin 25°} + \frac{1}{2}(1 + \cos\beta)\right]$$

With try and error: E_{Gen_Max} = 935.1 W/m² at $\beta = \beta_{Opt} \approx 55$ °

d) Extreme value consideration: Differentiate $E_{Gen} = f(\beta)$:

$$\frac{dE_{Gen}}{d\beta} = \frac{d}{d\beta}\left\{E_{Direct\_H} \cdot \left[\frac{\sin(\gamma_S + \beta)}{\sin\gamma_S} + \frac{1}{2}(1 + \cos\beta)\right]\right\} = E_{Direct\_H} \cdot \left[\frac{\cos(\gamma_S + \beta)}{\sin\gamma_S} - \frac{1}{2}(\sin\beta)\right]$$

$$\Rightarrow \frac{\cos(\gamma_S + \beta_{Opt})}{\sin\gamma_S} - \frac{1}{2}(\sin\beta_{Opt}) \overset{!}{=} 0$$

with addition theorem follows:

$$\Rightarrow \frac{\cos\gamma_S \cdot \cos\beta_{Opt} - \sin\gamma_S \cdot \sin\beta_{Opt}}{\sin\gamma_S} - \frac{1}{2} \cdot \sin\beta_{Opt} = 0 \Rightarrow \frac{\cos\gamma_S \cdot \cos\beta_{Opt}}{\sin\gamma_S} - \sin\beta_{Opt} - \frac{1}{2} \cdot \sin\beta_{Opt} = 0$$

$$\Rightarrow \frac{\cos\beta_{Opt}}{\tan\gamma_S} - \frac{3}{2} \cdot \sin\beta_{Opt} = 0 \Rightarrow \frac{\cos\beta_{Opt}}{\tan\gamma_S} = \frac{3}{2} \cdot \sin\beta_{Opt}$$

$$\Rightarrow \tan\beta_{Opt} = \frac{2}{3 \cdot \tan\gamma_S} \Rightarrow \beta_{Opt} = \arctan\left(\frac{2}{3 \cdot \tan\gamma_S}\right)$$

$$\Rightarrow \beta_{Opt} = \arctan\left(\frac{2}{3 \cdot \tan 25°}\right) = \underline{55.029°}$$

$\Rightarrow E_{Gen\_Max}$ = 935.11 W/m²